HEAT TRANSFER IN LONGITUDINAL LAMINAR FLOW OVER A BODY OF REVOLUTION

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This paper presents results of a numerical investigation of heat transfer in the example of longitudinal flow of a liquid at low Reynolds number (Re < 0.4) over a cylinder.

Some aspects of the design of technical equipment, e.g., for crystallization processes, are associated with the study of heat transfer during motion of solid particles in a liquid. Here one must know the surface distribution of heat transfer in addition to the integrated heat-transfer effect for the particle as a whole.

The data available in the literature mainly relate to bodies of spherical shape. The present paper studies heat transfer to bodies of cylindrical shape in a system with low Reynolds number (Re < 0.4).

Basic Equations and Boundary Conditions. To solve this problem the original equations are written in a cylindrical coordinate system fixed in the body, and are transformed to reflect the steady-state conditions and the flow symmetry (Fig. 1). The Navier–Stokes equations are

$$V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{\partial^2 V_r}{\partial z^2} - \frac{V_r}{r^2}\right), \tag{1}$$

$$V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2}\right). \tag{2}$$

The convective heat-transfer equation is

$$V_r \frac{\partial T}{\partial r} + V_z \frac{\partial T}{\partial z} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right). \tag{3}$$

The continuity equation is

$$\frac{\partial}{\partial r}(rV_r) + \frac{\partial}{\partial z}(rV_z) = 0. \tag{4}$$

We solve the system (1)-(4) for the following boundary conditions, on the cylinder surface (for $z = 0$, $0 \le r \le R_0$; $z = H$, $0 \le r \le R_0$; $0 \le z \le H$, $r = R_0$)

$$V_r = V_z = 0, \tag{5}$$

$$T = T_i; \tag{6}$$

far from the cylinder the liquid velocity is that of the oncoming stream (for $r \to \infty$, $z \to \pm\infty$)

$$V_r = 0,\ V_z = V_0, \tag{7}$$

$$T = T_0; \tag{8}$$

and on the axis of symmetry, which coincides with the z axis (for $z < 0$, $z > H$),

$$V_r = 0,\ \left.\frac{\partial V_r}{\partial r}\right|_{r=0} = 0. \tag{9}$$

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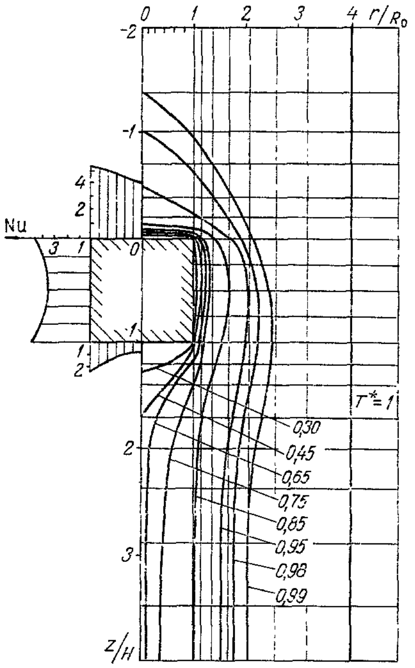

Fig. 1. Reduced temperature (T^*) and heat-transfer intensity (Nu) for Re = 0.2, Pr = 400, R_0/H = 0.5.

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0. \tag{10}$$

Numerical Solution. We write the equations of the original system, (1)-(3), in a form convenient for numerical integration. According to [3], these can be written as follows:

$$r^2\left\{\frac{\partial}{\partial z}\left(\frac{\omega}{r}\frac{\partial\Psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\frac{\omega}{r}\frac{\partial\Psi}{\partial z}\right)\right\}=\nu\left\{\frac{\partial}{\partial r}\left[r^3\frac{\partial}{\partial r}\left(\frac{\omega}{r}\right)\right]+\frac{\partial}{\partial z}\left[r^3\frac{\partial}{\partial z}\left(\frac{\omega}{r}\right)\right]\right\}, \tag{11}$$

$$\omega=-\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial\Psi}{\partial z}\right)-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right), \tag{12}$$

$$\frac{\partial}{\partial z}\left(T\frac{\partial\Psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(T\frac{\partial\Psi}{\partial z}\right)=a\left\{\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(r\frac{\partial T}{\partial z}\right)\right\}. \tag{13}$$

Here the boundary conditions will be as follows: On the cylinder surface (for $z = 0$, $0 \le r \le R_0$; $z = H$, $0 \le r \le R_0$; $0 \le z \le H$, $r = R_0$)

$$\Psi=0,\ T=T_i,\ \omega=-\frac{\partial}{\partial n}\left(\frac{1}{r}\frac{\partial\Psi}{\partial n}\right); \tag{14}$$

far from the cylinder (for $r \to \infty$, $z \to \pm\infty$)

$$\Psi=\frac{1}{2}r^2V_0,\ \omega=0,\ T=T_0; \tag{15}$$

and on the axis of symmetry (for $z < 0$, $z > H$)

$$\Psi=0,\ \omega=0,\ \left.\frac{\partial T}{\partial r}\right|_{r=0}=0. \tag{16}$$

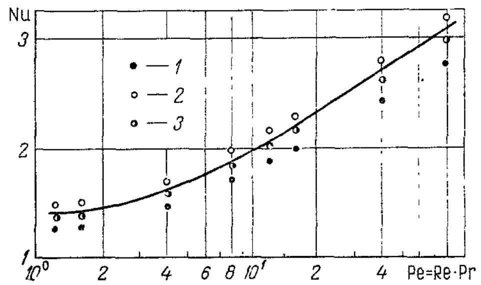

Fig. 2

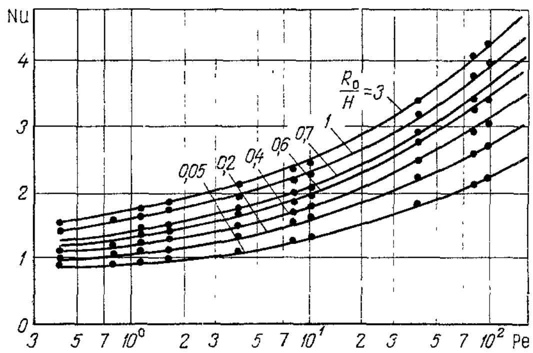

Fig. 3

Fig. 2. Effect of method of subdivision of the computing region on the accuracy of the numerical calculations. $R_0/H = 0.5$; Pe = RePr: 1) 11 × 15; 2) 18 × 25; 3) 20 × 28; the curve shows a 15 × 20 mesh (Fig. 1).

Fig. 3. The Nusselt number (Nu) as a function of Peclet number (Pe): The points show the results of numerical calculations and the curves are the theory of Eq. (19).

The transformation from system of equations (11)-(16) to the equations for computer calculation has been described in detail in [1, 3].

The program was implemented in the form of subroutines in ALGOL-60 relative to the TA-1M translator. The calculation was performed on the M-222 computer. The computing region was chosen, based on preliminary calculations, to exclude the influence of its size on the results obtained (Fig. 1). An estimate of the effect of the method of subdivision of the computing region on the accuracy of the numerical calculations was made. An increase in the number of mesh nodes by a factor of three had practically no effect on the accuracy of these calculations, the divergence was 10% (Fig. 2), and the transition from a nonuniform mesh to a uniform mesh had a negative effect (a 20 × 28 mesh).

From the isotherm we calculated the heat-transfer distribution over the particle surface (Fig. 1), according to [4]:

$$\mathrm{Nu} = -2\left.\frac{\partial T^*}{\partial n^*}\right|_{n^*=0} \tag{17}$$

The heat-transfer intensity for the particle as a whole was determined from the equation

$$\overline{\mathrm{Nu}} = \frac{1}{F_{\mathrm{cyl}}}\int\limits_{F_{\mathrm{cyl}}} \mathrm{Nu}\, dF. \tag{18}$$

Computational Results. The results of the numerical calculation for various ratios of the cylinder radius (R_0) to its height (H) are shown in Fig. 3. The calculated data can be approximated by the equation

$$\mathrm{Nu} = \mathrm{Nu}_0 f\,(R_0/H), \tag{19}$$

where Nu_0 is the heat-transfer intensity for a particle of spherical shape [2], and

$$\mathrm{Nu}_0 = 2 + 1.28\,\mathrm{Re}^{1/3}\,\mathrm{Pr}^{1/3}; \tag{20}$$

and $f(R_0/H)$ is a function accounting for the influence of particle shape.

A graphical interpretation of the function $f(R_0/H)$ is shown in Fig. 4, where one can see that the heat-transfer intensity increases with increase of R_0/H.

The range of variation of values of Re and Pr was limited for the specific application of the results obtained in estimating the influence of a crystal shape on its rate of growth from solutions and melts. Therefore, the Re value was varied in the range $10^{-3} \le \mathrm{Re} \le 20$ (no solutions were obtained for large Re), and of Pr in the range $50 \le \mathrm{Pr} \le 400$. The results, obtained for $0.4 \le \mathrm{Re} \le 20$, do not agree with Eq. (19). This is related to

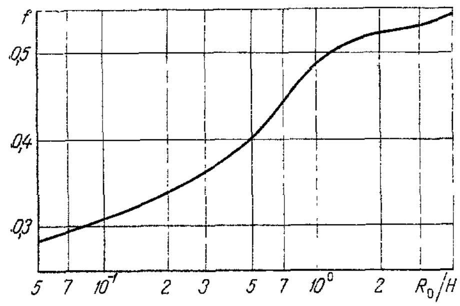

Fig. 4. The correction function f as a function of R_0/H.

a change in the cylinder flow conditions, i.e., with formation of a vortex zone. Thus, Eq. (19) can be recommended for calculating heat and mass transfer processes for Re $\leq$ 0.4 and Pr $\gg$ 1.

NOTATION

r, z	are the cylindrical coordinates;
R_0	is the cylinder radius;
H	is the cylinder height;
F_{cyl}	is the cylinder surface area;
n	is the distance from the cylinder surface along the normal;
n^*	is the distance from the cylinder surface along the normal, referenced to one of its linear dimensions;
V_r	is the radial liquid velocity component;
V_z	is the axial liquid velocity component;
V_0	is the velocity of the incident flow;
P	is the pressure;
ρ	is the density;
ν	is the kinematic viscosity;
a	is the thermal diffusivity;
T	is the temperature;
T_0	is the incident flow temperature;
T_i	is the temperature at the cylinder surface;
T^*	is the reduced temperature, $T^* = (T - T_0)/(T_i - T_0)$;
Ψ	is the stream function;
ω	is the vorticity;
Nu	is the Nusselt number;
Nu_0	is the Nusselt number for a particle of spherical range;
$Re = R_0V_0/\nu$	is the Reynolds number;
$Pr = \nu/a$	is the Prandtl number;
$Pe = R_0V_0/a$	is the Peclet number.

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